May 28 Math 2254 sec 001 Summer 2015

Section 6.4: Volume of a Solid by Slicing[1]

We'll call an object a **cylinder** if cross sections taken with respect to some axis are identical.

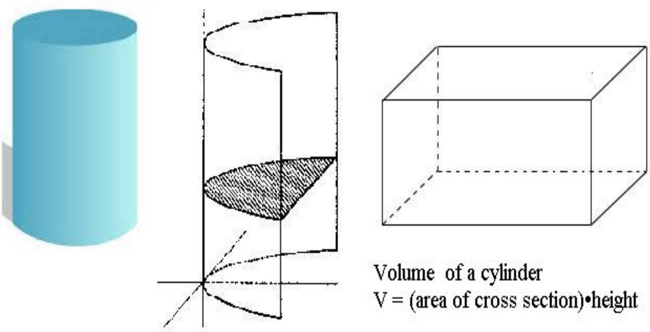


Figure: A circular, a parabolic, and a rectangular cylinder.

[1] We'll cover 6.4, then 6.2 and 6.3.

Volume of Solid by Slicing

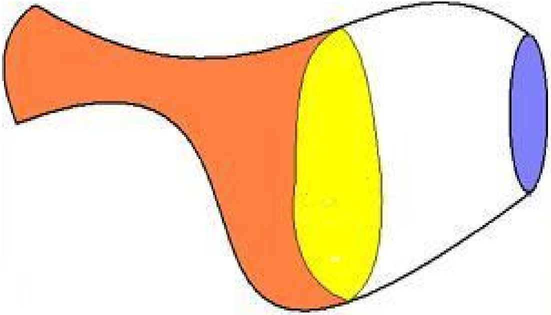


Figure: Suppose we have a solid that isn't actually a cylinder.

Volume of Solid by Slicing (Motivated by Bread)

Figure: Suppose we wish to find the volume of a loaf of bread.

Volume of Solid by Slicing (Motivated by Bread)

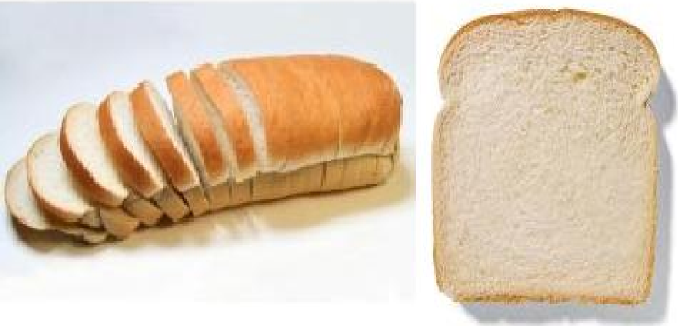


Figure: We can slice the loaf into pieces, and add the volumes of the slices.

Volume of Solid by Slicing

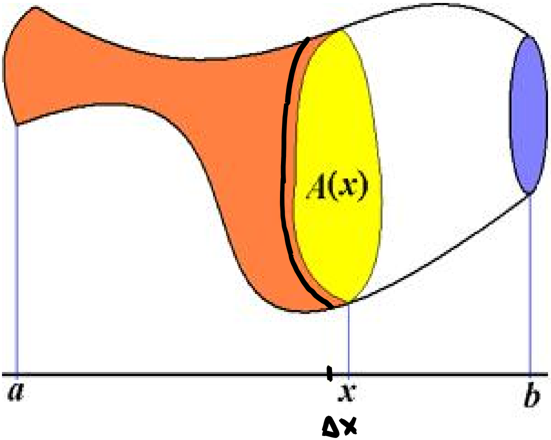


Figure: We place our solid over an x-axis for $a \leq x \leq b$. And consider a cross section at some x value.

Volume of Solid by Slicing

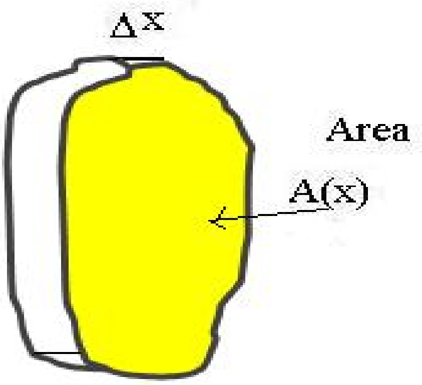


Figure: If we take slices at x and $x + \Delta x$ and remove a piece, it is *nearly* a cylinder of volume $V = A(x)\Delta x$.

Volume of Solid by Slicing

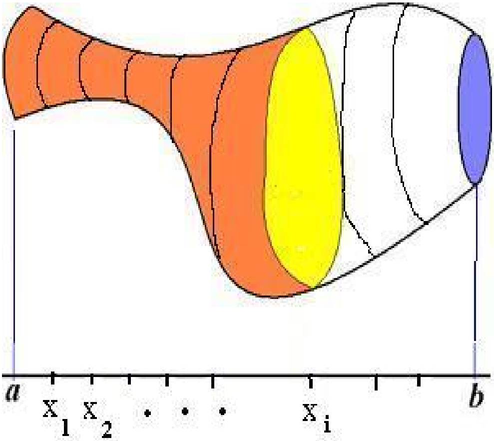


Figure: The total volume $V \approx A(x_1)\Delta x + A(x_2)\Delta x + \cdots + A(x_n)\Delta x$.

Volume of Solid by Slicing

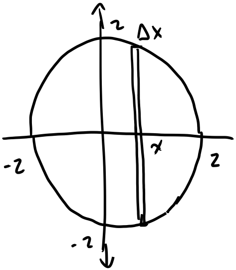
Our volume

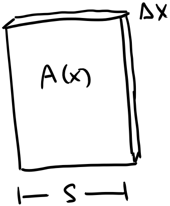
$$V \approx A(x_1)\Delta x + A(x_2)\Delta x + \cdots + A(x_n)\Delta x = \sum_{i=1}^{n} A(x_i)\Delta x$$

Volume: Let S be a solid that that lies between $x = a$ and $x = b$ having cross sectional area $A(x)$, where the cross section is in the plane through the solid perpendicular to the x-axis at each x in (a, b). The volume of S is

$$V = \lim_{n\to\infty} \sum_{i=1}^{n} A(x_i)\Delta x = \int_a^b A(x)\,dx.$$

An object has as its base the disk $x^2 + y^2 \leq 4$ in the xy-plane. Cross sections taken perpendicular to the x-axis are squares with one side in the plane. Find the volume of the solid.





s = side length

height Δx

$A(x) = ?$ $\quad A(x) = s^2$

$s = \sqrt{4-x^2} - \left(-\sqrt{4-x^2}\right)$

$s = 2\sqrt{4-x^2}$

$x^2+y^2=4 \implies y = \sqrt{4-x^2}$

or $y = -\sqrt{4-x^2}$

▸ Volume by Cross Section Applet

So $A(x) = \left(2\sqrt{4-x^2}\right)^2 = 4(4-x^2) = 16-4x^2$

Volume of one slice $V_{slice} = A(x)\,\Delta x = (16-4x^2)\,\Delta x$

add the slices from $x=-2$ to $x=2$ taking

$\Delta x \to dx$

$$V = \int_{-2}^{2} (16-4x^2)\,dx$$

Note

$A(x) = 16-4x^2$

is even

$$= 2\int_0^2 (16-4x^2)\,dx$$

using even symmetry

$$= 2\left[16x - \frac{4x^3}{3}\right|_0^2$$

$$= 2\left[16\cdot 2 - \frac{4(2^3)}{3} - 0\right]$$

$$= 64 - \frac{64}{3} = 64\left(1-\frac{1}{3}\right) = 64\left(\frac{2}{3}\right) = \frac{128}{3}$$