TRANSLATION LENGTHS OF POINT-PUSHING PSEUDO-ANOSOV MAPS OF RIEMANN SURFACES

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Abstract

Let S be a hyperbolic Riemann surface of genus $p > 1$, which has no free boundary curve and contains only one puncture x. Let $\mathcal{C}(S)$ denote the curve complex of S, which is equipped with a path metric $d_{\mathcal{C}}$. Let $\mathcal{C}_0(S)$ denote the set of vertices of $\mathcal{C}(S)$. Masur–Minsky showed that there exists a constant $c > 0$ such that for any pseudo-Anosov map $f : S \to S$, any integer n and $u \in \mathcal{C}_0(S)$, we have $d_{\mathcal{C}}(u, f^n(u)) > c|n|$.

In this presentation, we consider the subgroup $\mathscr{F}$ of the mapping class group $\mathrm{Mod}(S)$ which consists of elements isotopic to the identity on $\tilde{S} = S \cup \{x\}$. By a result of Kra, $\mathscr{F}$ contains infinitely many pseudo-Anosov elements and every pseudo-Anosov element in $\mathscr{F}$ determines an oriented filling closed geodesic on $\tilde{S}$. We prove the following result:

Theorem. *For any $f \in \mathscr{F}$, there exists $u \in \mathcal{C}_0(S)$ such that*

$$d_{\mathcal{C}}\left(u, f^m(u)\right) \geq \begin{cases} |m| & \text{if } |m| \leq 7, \\ \frac{2|m|+5}{3} & \text{if } |m| > 7. \end{cases}$$

Remarks. Set $\tau_{\mathcal{C}}(f) = \liminf_{m\to\infty} d_{\mathcal{C}}\left(u, f^m(u)\right)/m$. We know that $\tau_{\mathcal{C}}(f)$ depends only on f and is called the asymptotic translation length for the action of f on $\mathcal{C}(S)$. For a subgroup $H \subset \mathrm{Mod}(S)$ we write

$$L_{\mathcal{C}}(H) = \inf\left\{\tau_{\mathcal{C}}(f) : \text{ for all pseudo-Anosov elements in } H\right\}.$$

It is well known that as the genus $p \to +\infty$, $L_{\mathcal{C}}\left(\mathrm{Mod}(S)\right) \to 0$ and, for certain subgroups G of $\mathrm{Mod}(S)$, $L_{\mathcal{C}}(G) \to 0$. The question arises as to whether there are subgroups G' of $\mathrm{Mod}(S)$ such that $L_{\mathcal{C}}(G')$ does not tend to zero. As an immediate corollary of the above Theorem, we estimate that $\frac{2}{3} \leq L_{\mathcal{C}}(\mathscr{F}) \leq 1$ which answers the question negatively.

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